Name:

Linear Algebra (Math 2753)
Practice Exam # 1

Professor Paul Bailey
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This practice examination is essentially last year's midterm. It contains five problems which are worth 20 points each. The bonus problem is worth 10 additional points.

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus	Total Score

Problem 1. Multiple Choice
Circle the letter corresponding to the best answer.

Let $f : X \to Y$ be a function, with $A, B \subset X$ and $C, D \subset Y$. Which of the following is NOT always true?

(a) $f(A \cup B) = f(A) \cup f(B)$

(b) $f(A \cap B) = f(A) \cap f(B)$

(c) $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$

(d) $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$

The locus in $\mathbb{R}^3$ of the equation $z = x^2 + y^2$ is a

(a) sphere

(b) paraboloid

(c) cone

(d) plane

Let v and w be vectors in $\mathbb{R}^3$. The vector which proceeds from the tip of v to the tip of w is

(a) $v + w$

(b) $v - w$

(c) $w - v$

(d) $v \cdot w$

Given two planes in $\mathbb{R}^4$, which of the following is NOT possible:

(a) they intersect in a point

(b) they intersect in a line

(c) they intersect in a plane

(d) all are possible

Let A and B be finite subsets of $\mathbb{R}^n$. Which of the following is NOT always true:

(a) $\text{span}(A \cup B) = \text{span}(A) \cup \text{span}(B)$

(b) $\text{span}(A \cap B) = \text{span}(A) \cap \text{span}(B)$

(c) $A \subset B \Rightarrow \text{span}(A) \subset \text{span}(B)$

(d) all are always true

Problem 2. (Vectors)
Perform the indicated computation.

(a) Let $v = (1, 1)$ and $w = (1, y)$ be vectors in $\mathbb{R}^2$.
Find all values for y such that the angle between v and w is $30°$.

(b) Let $P = (2, 0, 2)$, $Q = (1, 3, -1)$, and $R = (-1, 0, 3)$ be points in $\mathbb{R}^3$.
Find the area of the triangle $\triangle OPQ$.

Problem 3. (Line and Planes)
Perform the indicated computation.

(a) Find the distance in $\mathbb{R}^3$ between the plane $\mathcal{H} : 6x - 2y + 9z = 5$ and the point $P : (1, -3, 2)$.

(b) Let $P : (1, 3, 1)$, $Q : (3, -1, 1)$, and $R(-1, 1, 3)$ be three points in $\mathbb{R}^3$. The set of all points in $\mathbb{R}^3$ which are equidistant to P, Q, and R is a line. Find the parametric equations for this line.

Definition 1. Let $V \subset \mathbb{R}^n$. We say that V is a *subspace* of $\mathbb{R}^n$, and write $V \leq \mathbb{R}^n$, if

(S0) V is nonempty;

(S1) $v, w \in V \Rightarrow v + w \in V$;

(S2) $v \in V, a \in \mathbb{R} \Rightarrow av \in V$.

Problem 4. (Subspaces)
Let $x, y \in \mathbb{R}^n$, and set

$$V = \{v \in \mathbb{R}^n \mid v \perp x \text{ and } v \perp y\}.$$

(a) Show that $V \leq \mathbb{R}^n$.

(b) Suppose that $n = 3$, $x = (1, -2, 3)$ and $y = (4, 2, -1)$. Find a basis for $V \leq \mathbb{R}^3$.

Definition 2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$. We say that T is a *linear transformation* if

(T1) $T(v + w) = T(v) + T(w)$, where $v, w \in \mathbb{R}^n$;

(T2) $T(av) = aT(v)$, where $v \in \mathbb{R}^n$ and $a \in \mathbb{R}$.

Problem 5. (Linear Transformations)
Consider the function

$$T : \mathbb{R}^3 \to \mathbb{R}^3 \quad \text{given by } T(x, y, z) = (y, x, -z).$$

(a) Show that T is a linear transformation.

(b) Find a vector $v \in \mathbb{R}^3$ such that $T(v) = v$.

(c) Describe the geometric effect of T.

Problem 6. (Bonus)
Let $d, e \in \mathbb{R}$ and set

$$E = \begin{bmatrix} 1 & 0 & 0 \\ e & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{bmatrix}, \quad P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

(a) Compute EA, DA, and PA.

(b) Describe in words the effect on $\mathbb{R}^3$ of the linear transformation corresponding to E, D, and P. Remember, the columns are the destinations of the standard basis vectors.